

# Neutrino oscillations: another physics?

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It is shown that the neutrino oscillations phenomenon may be attributed to the Wilson fermion doubling phenomenon. The Wilson fermion doubling exists only on the lattices, both periodic and non-periodic (simplicial complexes). Just the last case plays a key role here. Thereby, the neutrino oscillations may show for the existence of a space-time granularity.

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**1.** The neutrino oscillations, i.e. the mutual oscillating transitions of the neutrinos of different generations, are observed for a long time now. The common explanation of the phenomenon is based on the assumption the neutrino mass matrix is non-diagonal. Moreover, in order to match all the experimental evidences, it is necessary to introduce the extra neutrino fields, which are sterile regarding to all interactions (naturally, except gravitational one). The sterile neutrinos cannot be observed directly: they are coupled to the three known neutrino generations only by means of a common mass matrix, and this is the way they give a contribution to the neutrino oscillations. The introduction of sterile neutrinos does not exhaust all difficulties of the theory. The detailed description of the neutrino oscillations experiments and theory can be found, for example, in [1], [2], and in numerous references there.

It is shown here that another physics may provide the neutrino oscillations. The basis for this physics is the Wilson lattice fermion doubling phenomenon [3–8]. Here the Wilson doubling on irregular lattices (simplicial complexes) is interesting (see below). This case was studied in [9], [10]. Thus first of all I must outline shortly the physics of irregular (doubled) fermion quanta on the "breathing" simplicial complexes [10] in the frame of discrete gravity (see also [11]).

**2.** Let's describe shortly the model of discrete quantum gravity which is determined on the simplicial complexes. For simplicity, only the gravitational and fermionic dynamic variables are introduced in this model. Since the mutual geometrical locations of the vertices of the complex are determined by the gravitational variables, the mutual geometrical locations of the vertices are described by a wave function in the quantum the-

ory. Therefore the simplicial complex can be named as "breathing" one here.

Note that the considered model of discrete quantum gravity is not realistic for some reasons. Nevertheless, maybe certain features of the model adequately simulate a Nature character. In this work I make the assumption that the fermion fields are determined on a "breathing" 4-dimensional simplicial complex. This assumption is basic here.

Further all definitions and designations are similar to that in [10, 11]. The quantities related with Euclidean signature are supplied by the additional lower index ( $E$ ). For example, the Euclidean Dirac matrices ( $4 \times 4$ )

$$\begin{aligned} \gamma_{(E)}^a \gamma_{(E)}^b + \gamma_{(E)}^b \gamma_{(E)}^a &= 2\delta^{ab}, \quad \gamma_{(E)}^5 = \gamma_{(E)}^1 \gamma_{(E)}^2 \gamma_{(E)}^3 \gamma_{(E)}^4, \\ \text{tr } \gamma_{(E)}^5 \gamma_{(E)}^a \gamma_{(E)}^b \gamma_{(E)}^c \gamma_{(E)}^d &= 4 \varepsilon_{(E)}^{abcd}. \end{aligned} \quad (1)$$

For each oriented 1-simplex  $a_i a_j$  of the 4-dimensional simplicial complex  $\mathfrak{K}$  an element of the group  $\text{Spin}(4)$

$$\Omega_{ij} = \Omega_{ji}^{-1} = \exp \left( \frac{1}{2} \omega_{(E)ij}^{ab} \sigma_{(E)}^{ab} \right), \quad \sigma_{(E)}^{ab} = \frac{1}{4} [\gamma_{(E)}^a, \gamma_{(E)}^b] \quad (2)$$

and an element of the Clifford algebra

$$\hat{e}_{(E)ij} \equiv e_{(E)ij}^a \gamma_{(E)}^a \equiv -\Omega_{ij} \hat{e}_{(E)ji} \Omega_{ij}^{-1}, \quad (3)$$

are assigned. The elements (2) belong to the compact group if the variables  $\omega_{(E)ij}^{ab}$  are real. Let the index  $A$  enumerates 4-simplices,  $a_{Ai}, a_{Aj}, a_{Ak}, a_{Al}$ , and  $a_{Am}$  be all five vertices of a 4-simplex with index  $A$  and  $\varepsilon_{Aijklm} = \pm 1$  depending on whether the order of vertices  $a_{Ai} a_{Aj} a_{Ak} a_{Al} a_{Am}$  defines the positive or negative orientation of this 4-simplex. Later the notations  $a_{Ai}, a_{Aj}, \Omega_{Aij}$  and so on indicate that 1-simplex  $a_i a_j$  belong to 4-simplex with index  $A$ . The action of the gravitational and Dirac fields associated with four-dimensional

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simplicial complex  $\mathfrak{K}$  has the form

$$\mathfrak{A} = \frac{1}{5 \times 24} \sum_A \sum_{i,j,k,l,m} \varepsilon_{Aijklm} \text{tr } \gamma_{(E)}^5 \times \\ \times \left\{ \frac{2}{l_P^2} \Omega_{Ami} \Omega_{Aij} \Omega_{Ajm} \hat{e}_{(E)Amk} \hat{e}_{(E)Aml} - \right. \\ \left. - \frac{1}{24} \hat{\Theta}_{Ami} \hat{e}_{(E)Amj} \hat{e}_{(E)Amk} \hat{e}_{(E)Aml} \right\}, \quad (4)$$

$$\hat{\Theta}_{Aij} = \frac{1}{2} \gamma_{(E)}^a \left( \psi_{Ai}^\dagger \gamma_{(E)}^a \Omega_{Aij} \psi_{Aj} - \psi_{Aj}^\dagger \Omega_{Aji} \gamma_{(E)}^a \psi_{Ai} \right). \quad (5)$$

The partition function for a discrete gravity is defined as follows:

$$\mathfrak{U} \sim \left( \prod_{\mathcal{E}} \int d\Omega_{\mathcal{E}} \int d\epsilon_{\mathcal{E}} \right) \left( \prod_{\mathcal{V}} \int d\psi_{\mathcal{V}}^\dagger d\psi_{\mathcal{V}} \right) \exp(i \mathfrak{A}). \quad (6)$$

The indexes  $\mathcal{V}$  and  $\mathcal{E}$  enumerate all vertexes and 1-simplexes, correspondingly,  $d\Omega_{\mathcal{E}}$  is the Haar measure on the group  $\text{Spin}(4)$ ,  $d\epsilon_{\mathcal{E}} = \prod_a d\epsilon_{\mathcal{E}}^a$ .

Let's observe the formal transition to the continuum physics with Minkowski signature. To pass to the Minkowski signature one must deform the integration paths in (6) and make the substitutions as follows ( $\alpha, \beta, \dots = 1, 2, 3$ )

$$\omega_{(E)ij}^{4\alpha} = i \omega_{ij}^{0\alpha}, \quad \omega_{(E)ij}^{\alpha\beta} = -\omega_{ij}^{\alpha\beta}, \\ e_{(E)ij}^4 = i e_{ij}^0, \quad e_{(E)ij}^\alpha = -e_{ij}^\alpha, \\ \gamma_{(E)}^4 = \gamma^0, \quad \gamma_{(E)}^\alpha = i \gamma^\alpha, \quad \psi_{(E)i}^\dagger = \bar{\psi} = \psi^\dagger \gamma^0. \quad (7)$$

The variables  $\omega_{ij}^{ab}$  and  $e_{ij}^a$  in the right hand sides of Eqs. (7) are real. So we have

$$\omega_{(E)ij}^{ab} \sigma_{(E)}^{ab} = \omega_{ij}^{ab} \sigma_{ab}, \quad \sigma^{ab} \equiv \frac{1}{4} [\gamma^a, \gamma^b], \quad (8)$$

and the elements (2) of the compact group  $\text{Spin}(4)$  transform to the elements of the noncompact group  $\text{Spin}(3,1)$ . We introduce the local coordinates of the vertices [12]:  $x_{Ai}^\mu \equiv x^\mu(a_{Ai})$ , where  $\mu = 1, 2, 3, 4$  and  $dx_{ji}^\mu \equiv x_{Ai}^\mu - x_{Aj}^\mu$ . Suppose we have a smooth fields  $\omega_\mu^{ab}(x)$  and  $e_\mu^a(x)$  in the Euclidean space. Then the lattice variables  $\omega_{ji}^{ab} = \omega_\mu^{ab}(x) dx_{ji}^\mu$ ,  $e_{mi}^a = e_\mu^a(x) dx_{mi}^\mu$ ,  $x = x_{Aj}$  are determined by the 1-forms  $\omega_\mu^{ab}(x) dx_{ji}^\mu$  and  $e_\mu^a(x) dx_{ji}^\mu$ . The inverse procedure is considered in [10, 11]. The fermion field is a 0-form:  $\psi_i = \psi(x)$ ,  $x^\mu \equiv x_{Ai}^\mu$ . Thus, neglecting high derivatives of the fields  $\omega_{(E)\mu}^{ab}(x)$ ,  $e_{(E)\mu}^a(x)$  and  $\psi(x)$ , we pass on from the lattice action (4) to the usual continual Minkowski gravity action in the Palatini form up to the multiplier  $(-i)$ . This unnecessary multiplier is easily removed by the the substitution  $x^4 = i x^0$ ,

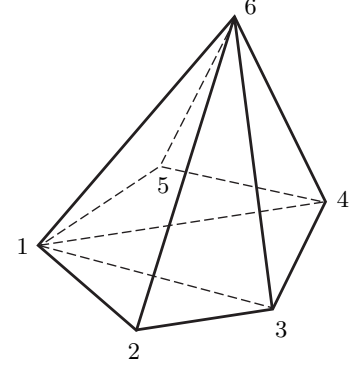


FIG. 1: An example of 2-dimensional complex with the fermion doubling property

where the coordinate  $x^0$  is the real Minkowski time. Indeed, the action in Palatini form is the integral of 4-form which is linear and uniform relative to the  $dx^4$ . Thus in the naive long-wave limit and Minkowski signature we have the usual gravitational action in Palatini form.

**3.** Now I discuss shortly the problem of Wilson fermion doubling phenomenon on the irregular lattice [9, 10]. For that end the situation must be simplified as much as possible. Thus further we believe that

$$\Omega_{ij} = 1, \quad (e_{ij}^a + e_{jk}^a + \dots + e_{li}^a) = 0. \quad (9)$$

Here the sum in the parentheses is taken on any closed path. Eqs. (9) mean that the curvature and torsion are equal to zero, so that the geometrical realization of the  $n$  Dimensional complex  $\mathfrak{K}$  is in the  $n$  Dimensional Minkowski space, the cartesian coordinates of a vertex  $a_i$  have the values  $x_i^a$  and  $e_{ij}^a = x_j^a - x_i^a$ ,  $a = 0, \dots, n-1$ . Here we are interested in the cases  $n = 4, 3$ .

We need the lattice analog of the Dirac Hamiltonian. For this let's choose a spacelike  $(n-1)D$  subcomplex  $\mathfrak{S}$  of the  $nD$  complex  $\mathfrak{K}$ . It is assumed that the geometrical realization of the subcomplex  $\mathfrak{S}$  is in the hypersurface determined by the Eq.  $x^0 = \text{const}$ . Thus, we have all  $e_{ij}^0 = 0$  on  $\mathfrak{S}$ , and instead of the  $e_{ij}^a$  the quantities  $e_{pr}^\alpha$  are used. The vertices of subcomplex  $\mathfrak{S}$  are designated as  $a_p, a_q, a_r, \dots$ . Equation for the zero eigenfunction of the Dirac Hamiltonian looks like

$$-\frac{i}{(n-1)} \sum_{q(p)} \sum_{\alpha} S_{\alpha pq} \gamma^0 \gamma^\alpha \psi_q^{(0)} = 0, \quad (10)$$

$$S_{\alpha pq} = \frac{1}{((n-2)!)^2} \sum_{A(p,q)} \sum_{\{r_1, \dots, r_{n-2}\}} \varepsilon_{\alpha \beta_1 \dots \beta_{n-2}} \times \\ \times \varepsilon_{A(p,q) p q r_1 \dots r_{n-2}} e_{A(p,q) p r_1}^{\beta_1} \dots e_{A(p,q) p r_{n-2}}^{\beta_{n-2}} \quad (11)$$

Here index  $A(p,q)$  enumerates all  $(n-1)$ -simplexes containing a fixed 1-simplex  $a_p a_q$ , index  $q(p)$  enumerates all vertexes  $a_q$  of the 1-simplexes  $a_p a_q$  with a fixed vertex

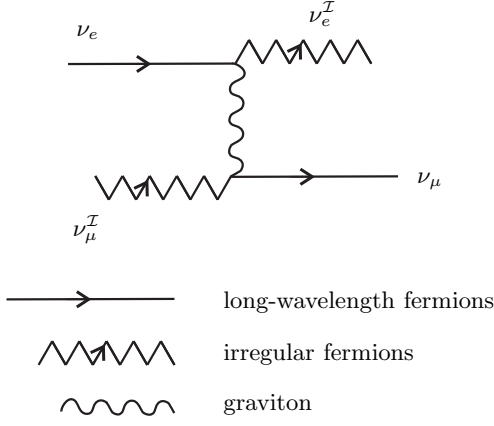


FIG. 2: The process of the electron neutrino transition to the muon one.

$a_p, \varepsilon_{A(p,q)pqr_1 \dots r_{n-2}} = \pm 1$  depending on the orientation of the  $(n-1)$ -simplex  $a_p a_q a_{r_1} \dots a_{r_{n-2}}$ .

For simplicity let's consider the case  $n = 3$ . Let  $\mathbf{v}_p$  denotes the part of the subcomplex  $\mathfrak{S}$  consisting of all 2-simplexes having a common vertex  $a_p$ . We enumerate the vertexes  $a_{q(p)} \in \partial \mathbf{v}_p$  so that the vertex  $a_{q(p)+1}$  go after the vertex  $a_{q(p)}$  in tracing the boundary  $\partial \mathbf{v}_p$  anticlockwise, and we shall assume that the index  $q(p)$  is determined up to  $(\text{mod } n)$ , where  $n$  is the number of vertexes on  $\partial \mathbf{v}_p$ . Introduce the complex coordinate  $z_p = x_p^1 + ix_p^2$  for the vertex  $a_p$ . Let the spinor  $\psi$  be 2-component, the upper component is designated as  $\varphi$ ,  $\gamma^\alpha = \sigma^\alpha$ ,  $\alpha = 1, 2$ . Eq. (10) splits, and for upper component we have [9]:

$$\sum_{q(p)} z_{q(p)} (\varphi_{q(p)+1} - \varphi_{q(p)-1}) = 0. \quad (12)$$

The Wilson doubling means that the system of equations (12) has the irregular solutions  $\varphi_{p,q}^I \equiv \varphi_p^I - \varphi_q^I \neq 0$ ,  $|\varphi_{p,q}^I| \sim |\varphi_p^I|$  for the next vertexes  $a_p$  and  $a_q$ .

In [9] the 2-D complexes are constructed which do not possess the Wilson doubling property. But such a complexes are not to be seen as typical one, but most probably as exclusive complexes. For example, the 2-D complex with 6 vertexes and  $S^2$ -topology (see Fig. 1) possess the Wilson doubling property. Many other examples also demonstrate this property. Unfortunately, I have not classified the simplicial complexes with respect to the Wilson doubling property. But being guided by the experience I shall assume that almost all simplicial complexes manifest the Wilson doubling.

4. Let  $\{\psi_{s,j}^{I(0)}\}, s = 1, 2, \dots, S$  be the complete independent set of the irregular zero modes. Each of them satisfy Eq. (10) on a 3D subcomplex  $\mathfrak{S}$  and the discrete Dirac equation with Minkowski signature on the complex  $\mathfrak{K}$ . The soft irregular modes can be conceived of as  $\psi_j^I \{g\} = \sum_s g_{s,j}^I \psi_{s,j}^{I(0)}$ , where the fields  $g_{s,j}^I$  are slowly

varying and satisfying the equation

$$i \sum_{s'} \alpha_{i s s'}^b \partial_b g_{s' i}^I = 0, \quad (13)$$

$$\alpha_{i s s'}^b = \frac{1}{n(n+1)} \left[ \sum_a \sum_{j(i)} \left( \bar{\psi}_{s i}^{I(0)} S_{a i j} e_{i j}^b \gamma^a \psi_{s' j}^{I(0)} \right) \right]. \quad (14)$$

Here the quantity  $S_{a i j}$  is constructed in the same way as in (11) but on the 4D complex  $\mathfrak{K}$ . Now the irregular part of the Dirac field is expressed as

$$\psi_i^I = \sum_s g_{s i}^I \psi_{s i}^{I(0)}, \quad (15)$$

where the fields  $g_{s i}^I$  are the Grassmann variables. Here our main interest is the chronological correlator

$$i S_c^I(x_i - x_j) \equiv \langle \hat{T} \psi_i^I \bar{\psi}_j^I \rangle_{\psi, e} = \psi_{s i}^{I(0)} \langle (i \alpha^a \partial_a)^{-1}_{s s' i j} \rangle_e \bar{\psi}_{s' j}^{I(0)} \sim a^2 \gamma^a \partial_a \delta^{(4)}(x_i - x_j). \quad (16)$$

The subscripts  $\psi, e$  mean the averaging over Grassmann and  $e$ -variables, correspondingly. Here  $a$  acts as the effective lattice scale. It is seen from (16) that the irregular quanta are "bad" quasiparticles. This is a consequence of the facts that (i) the lattice is irregular and "breathing" and (ii) the irregular quanta interact strongly with the lattice. The result (16) is important below, it has been obtained in [10].

5. Let us suppose that there are two left neutrino fields  $\nu_e$  and  $\nu_\mu$  in the model [13]. Suppose also that there are the nonzero densities  $n_{\nu_e}^I$  and  $n_{\nu_\mu}^I$  of the irregular quanta (calculated per volume of quanta number).

Let's consider the scattering process pictured in Fig. 2. In Fig. 3 the same process is considered "under the microscope" (on the lattice). One must take into account that after the calculation of fermion integral in (6) the fermion segments with arrows will be assigned to 1-simplices with following properties: (i) the fermion segments form continuous broken lines on the complex, so that the arrows are unidirectional; (ii) two arrows of each kind of fermions come into each vertex and two arrows come out; (iii) each fermion broken line can be closed or unclosed. In the first case we have a vacuum fluctuation, in the latter case the line describes propagation of a real fermion.

It seems that the propagation of a real irregular fermion on the lattice is similar in a sense to the dynamics of a Brownian particle. But in our case, due to the chaotic interference, the propagator (16) decreases at  $|x_i - x_j| \rightarrow \infty$  more quickly than the probability of Brownian random walks for the distance  $|x_i - x_j|$ . This analogy once more justifies Eq. (16).

The symbols  $\odot$  and  $\circ$  in Fig. 3 mean the contribution from the gravity part of the action (4) under high-temperature expansion. For long-wavelength physics the

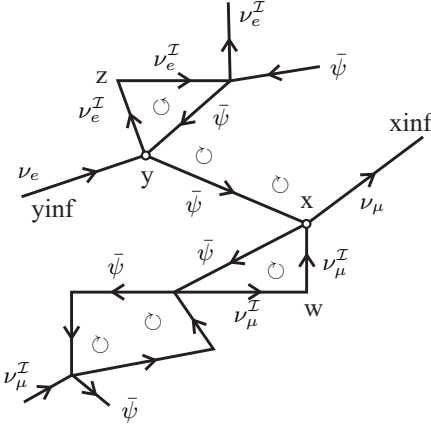


FIG. 3: The process of electron neutrino transition to the muon one “under the microscope” on the lattice

high-temperature expansion does not work. The lines ( $y_\infty \rightarrow y$ ) and ( $x \rightarrow x_\infty$ ) denote the long-wavelength quanta of  $\nu_e$  and  $\nu_\mu$  popagation, correspondingly.

Let’s estimate the amplitude of the process on Figs. 2, 3. Note that the asymptotic quanta  $\nu_e$  and  $\nu_\mu$  give the factor  $\{\exp(ik_e y - ik_\mu x) / \sqrt{\omega_e \omega_\mu}\}$  if the long-wavelength quanta are normalized as one quantum per unit volume. Here  $k$  and  $\omega$  are the 4-momentum and frequency of quanta. The irregular fermion quanta do not the asymptotic states, therefore they give the contribution  $\{iS_{c\mu}^I(x-w) \otimes iS_{ce}^I(y-z)\}$ . (The external spinor states are not interesting here.) Thus the estimation for the amplitude is as follows

$$iM \sim \int d^4x \int d^4y \frac{\exp(ik_e y - ik_\mu x)}{\sqrt{\omega_e \omega_\mu}} \times \{iS_{c\mu}^I(x-w) \otimes iS_{ce}^I(y-z)\} \{-iGD^I(x-y)\}. \quad (17)$$

Here  $G$  is the Newton gravitational constant,  $D^I(x-y)$  means the graviton propagator of irregular (not long-wavelength) quanta. The spinor or tensor indexes are not interesting here. Obviously, the estimation  $D^I(x-y) \sim a^2 \delta^{(4)}(x-y)$  is valid (compare with (16)). I shall assume that  $a \sim l_P$  and  $\delta^{(4)}(0) \sim l_P^{-4}$ , where  $l_P$  is the Planck scale,  $G = l_P^2$ . Since  $|w-x| \sim |y-z| \sim l_P$ , the integrations over  $y$  and  $x$  in (17) give

$$iM \sim \frac{1}{l_P^2 \sqrt{\omega_e \omega_\mu}} \int d^4x e^{i(k_e - k_\mu)x} \sim \frac{\Delta x^0 \delta^{(3)}(\mathbf{k}_e - \mathbf{k}_\mu)}{l_P^2 \omega}. \quad (18)$$

It is assumed that the translational invariance of the system is valid. Here  $\Delta x^0$  in (18) is of the order of the process duration,  $\Delta x^0 \sim l_P$ . Thus, omitting the  $\delta$ -function, we obtain

$$M \sim \frac{1}{l_P \omega}. \quad (19)$$

Note that in the most ultraviolet case, when  $\omega \sim l_P^{-1}$ , we have the natural result  $M \sim 1$ .

To obtain the change of the amplitude per unit time one should multiply the quantity (19) by the number of elementary events per unit time ( $n_{\nu_\mu}^I l_P^2$ ):

$$\frac{d}{dt} M \sim \frac{\hbar c^2 l_P}{\omega} n_{\nu_\mu}^I. \quad (20)$$

The comparison of the last result with the well known formulas (see [1], [2]) leads to the estimation

$$\frac{\hbar c^2 l_P}{\omega} n_{\nu_\mu}^I \sim \frac{\delta m^2 c^4}{\hbar \omega} \rightarrow n_{\nu_\mu}^I \sim \frac{c^2 \delta m^2}{\hbar^2 l_P} \sim 10^{42} \text{cm}^{-3}. \quad (21)$$

More likely, the estimation (21) is inflated by many orders.

Finally, we conclude that the neutrino oscillations should be observed since there are mutual transitions of the electron and muon neutrinos with fixed and equal momenta.

**6.** Instead of a Conclusion I would like to pose the question: Does the effect of neutrino oscillations give evidence in favour of the space-time granularity?

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  - [12] The construction of the local coordinates can be realized as follows. Let’s begin with an immersion of any 4-simplex to a four dimensional Euclidean space. Thus its vertexes acquire the Cartesian coordinates. Then the same is to be done with all adjacent 4-simplices, so that the common vertexes remain common in the Euclidean space, and so on. This process is possible since complex  $\mathfrak{R}$  is orientable.
  - [13] The model admits the introduction of any number of the neutrino massless fields.